(12)

AD-E 330 173

AD

AD A123703

TECHNICAL REPORT ARBRL-TR-02463

(Supersedes IMR No. 728)

A CRITIQUE OF THE BELL HELICOPTER TEXTRON COBRA 2.75 INCH ROCKET BALLISTIC ALGORITHM

Harold J. Breaux

January 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND

BALLISTIC RESEARCH LABORATORY

ABERDEEN PROVING GROUND, MARYLAND

Approved for public release; distribution unlimited.

046

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER TECHNICAL REPORT ARBRL-TR-02463	2. GOVT ACCESSION NO. AD-A123 203	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Critique of the Bell Helicopter Textron COBRA 2.75 Inch Rocket Ballistic Algorithm		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) HAROLD J. BREAUX		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Ballistic Research Laboratory ATTN: DRDAR-BLB Aberdeen Proving Ground, MD 21005		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Project No. 1H464202DL62
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Command US Army Ballistic Research Laboratory (DRDAR-BL) Aberdeen Proving Ground, MD 21005		12. REPORT DATE January 1983 13. NUMBER OF PAGES 33
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This report supersedes BRL IMR No. 728, dated Oct 1981.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

COBRA	Ballistic Algorithm
Helicopters	Fire Control
2.75 Inch Rockets	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The ballistic algorithm currently implemented on the AH-1S (Modernized) Cobra Helicopter for the 2.75 inch MK 40 rocket system is of interest due to ongoing efforts involving the MK 66 rocket system. Additional interest arises due to past unresolved controversies centering on the issue of the adequacy and correctness of the mathematical formulation. The question is examined by comparison of the Bell Helicopter Textron (BHT) algorithm with a simple free flight model derived from basic ballistic equations. The analysis indicates the

(Continued on reverse side)

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Item 20. ABSTRACT (continued)

omission of a significant drag effect in the elevation equation and the lack of any compensation for wind effects in free flight. The BHT algorithm is shown to be deficient in the formulation of a component term which compensates for the ballistic influence of helicopter velocity normal to the line of site. The general inaccuracy of the 2.75-inch rocket has previously masked any chance of detection of a systematic bias that might be due to a poor ballistic algorithm. The insight provided by this critique leads one to focus on hover firings where analysis predicts a bias should be more clearly evident. The test data provides a dramatic confirmation of the theoretical inferences which imply a 60 mil elevation error bias at maximum range. Simultaneously the analysis provides a basis for a mathematical cure for most of the observed deficiencies.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure | Page

LIST OF TABLES

I. INTRODUCTION

A ballistic algorithm for helicopter fire control is a collection of formulas and constants which in conjunction with sensors and an on-board computer serve to predict how to aim the aircraft guns and rockets. The ballistic algorithm for the 2.75 inch rockets as developed by Bell Helicopter Textron (BHT)[*1,2] is based on a modification and extension of equations developed by Clark[3] of BRL. The extension by BHT consisted primarily of addition of equation components to account for the influence of downwash. The BHT modifications included alterations made in the structure of the free flight expression for "gravity drop" and the addition of ad hoc terms to represent the effect of helicopter velocity normal to the line of sight References 1 and 2 list the implemented algorithm. However, there seems to be no documentation concerning the rationale for or effect on accuracy of these changes. O'Bryon of BRL[4] performed a numerical study of the initial modifications by BHT, Reference 1, and observed significant discrepancies between sight settings as predicted by BRL versus BHT equations. The subsequent BHT modifications contained in Reference 2 differ from Reference 1 mostly in the readjustment of constants. The major questions regarding the BHT equations and modifications concern their structure, in addition to the specific values of certain constants.

One of the major changes made by BHT consisted of representing gravity drop by an expression dependent on the square of time of flight (t^2), rather than a power series in range as derived by BRL.[3] A conversation between the author and Mr. Laird Taylor, formerly of BHT, in September 1980 indicates that the t^2 term was empirically based but was thought to be adequate.

In order to understand the effect of these changes and determine their validity (or not) a derivation from first principles is made herein. This derivation will show how a t^2 term can be justified but will also show how it yields limited accuracy when used alone (as in the BHT equations) rather than as part of the more complete ballistic approximation suggested by ballistic theory. Simultaneously the development will shed light on the additional ad hoc terms for representing the influence of vertical aircraft velocity. The development will also show the omission of the significant influence of free flight wind effects in the BHT algorithm.

---

*

1. *Bell Helicopter Textron Report No. 209-099-520 (07/20/78), "Modernized COBRA Fire Control Computer Mechanization Equations," Rev. B, 15 Aug 1978.*
2. *Ibid, Rev E, 22 Aug 1979.*
3. *Clark, David L., "A Sight Setting Equation for Air Launched Ballistic Rockets," unpublished BRL manuscript.*
4. *O'Bryon, James F., "Numerical Comparison of BHT Equation #209-099-520 With Comparable BRL Equation of the MK 40 2.75 Inch Rocket." Contained in a ltr dtd 20 Sep 1978 to PM COBRA (Mr. Ryan), SUBJECT: Fire Control Equation Data for 2.75 Inch Rockets for AH-1S Modernized COBRA.*

II. THE IDEALIZED FREE FLIGHT TRAJECTORY

A solution for an idealized free flight trajectory will be developed parallel to that obtained by the author, Ref. 5. Here, however, the coordinate system used by Clark and BHT will be employed. In Figure 1 the x axis is taken to be along the initial rocket line. The z axis is in a plane formed by the rocket line and the gravity vector. z is normal to x with the positive direction downward.

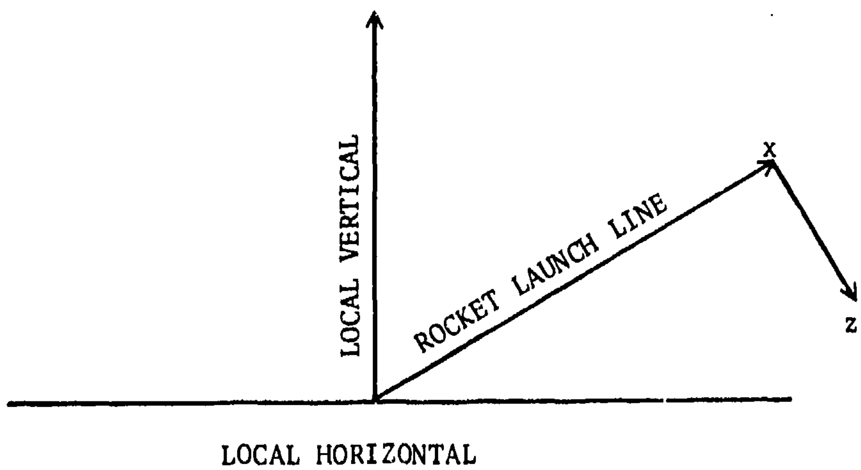

Figure 1. Coordinate system used in developing idealized free flight rocket trajectory.

By proceeding as in Ref. 5 the differential equations governing the idealized trajectory are given by

$$\ddot{x} = -\eta\,(\dot{x} - w_x)^2 + g_x \tag{1}$$

$$\ddot{z} = -\eta\,(\dot{x} - w_x)(\dot{z} - w_z) + g_z \quad . \tag{2}$$

Eqs. (1) and (2) are to be solved subject to the initial conditions

$$\begin{aligned} x &= x_b\,, & t &= t_b \\ z &= z_b, & t &= t_b \\ \dot{x} &= V_{xb}, & t &= t_b \\ \dot{z} &= V_{zb}, & t &= t_b \end{aligned} \tag{3}$$

5. *Breaux, Harold J., "A Methodology for the Development of Fire Control Equations for Guns and Rockets Fired From Aircraft," April 1982, Draft BRL Manuscript.*

Here the subscript b denotes burnout conditions. In Eqs. (1) and (2) η is a lumped parameter given by

$$\eta = \pi d^2 C_D (M, \alpha)/8 m \quad , \tag{4}$$

d is the rocket diameter, m the rocket mass and C_D (M, α) is the drag coefficient. M is the mach number and α is the angle of attack. The notation $C_D(M,\alpha)$ is used to indicate that C_D is evaluated for the launch mach number M and launch angle of attack α. C_D is thereby assumed constant leading to η also being constant. (NOTE: The complete methodology (not discussed here) provides a means to compensate for the error associated with this assumption.) To obtain a solution to Eqs. (1) and (2) one can proceed as in Ref. 5. Let

$$\zeta = x - x_b - w_x (t - t_b) \tag{5}$$

$$\xi = z - z_b - w_z (t - t_b) \quad . \tag{6}$$

After these transformations Eqs. (1) and (2) transform to

$$\ddot{\zeta} = - \eta \dot{\xi}^2 + g_x \tag{7}$$

$$\ddot{\xi} = - \eta \dot{\zeta} \dot{\xi} + g_z \quad . \tag{8}$$

The initial conditions transform to

$$\dot{\zeta}(t_b) = V_{xb} - w_x = v_{xb} \tag{9}$$

$$\zeta(t_b) = 0 \tag{10}$$

$$\dot{\xi}(t_b) = V_{zb} - w_z = v_{zb} \tag{11}$$

$$\xi(t_b) = 0 \quad . \tag{12}$$

To obtain the solution to Eq. (7) first introduce the transformation

$$\dot{\zeta} = [u(\zeta)]^{1/2} \quad . \tag{13}$$

Inserting Eq. (13) into (7) yields the transformed differential equation

$$du/d\zeta = -2\eta \ u + 2g_x, \tag{14}$$

which has the solution

$$u = g_x/\eta + u_{xb}^{*2} \ e^{-2\eta\zeta} \quad . \tag{15}$$

u^*_{xb} is given by

$$u_{xb}^{*\,2} = v_{xb}^2 - g_x/\eta \quad . \tag{16}$$

Since

$$v_{xb}^2 >> g_x/\eta \tag{17}$$

hereafter the solution will be written

$$u = g_x/\eta + v_{xb}^2 \ e^{-2\eta\zeta} \quad . \tag{18}$$

By employing Eq. (18) in (13) one obtains the solution for t

$$t-t_b = \int_o^\zeta [g_x/\eta + v_{xb}^2 \ e^{-2\eta\zeta}]^{-1/2} d\zeta = (\eta v_{xb})^{-1} \int_1^{e^{\eta\zeta}} [1+\lambda p^2]^{-1/2} \ dp \tag{19}$$

where

$$\lambda = g_x/(\eta v_{xb}^2) \ . \tag{20}$$

The integral in Eq. (19) is given by

$$t - t_b = (\eta v_{xb} \lambda^{1/2})^{-1} \ \ln \ [p\lambda^{1/2} + (\lambda \ p^2+1)^{1/2}] \Bigg|_1^{e^{\eta\zeta}} \quad . \tag{21}$$

Now observe that

$$\ln \ [q + (q^2+1)^{1/2}] = q-q^3/6 + 3q^5/40 - \ . \ . \ . \tag{22}$$

By use of Eq. (22) in (21) one obtains

$$t - t_b = (\eta v_{xb})^{-1}\{(e^{\eta\zeta} - 1) - (\lambda/6)(e^{3\eta\zeta} - 1)\} . \tag{23}$$

In seeking a solution for Eq. (8) we let

$$\xi' = d\xi/d\zeta \quad . \tag{24}$$

One has

$$\dot{\xi} = \xi'\dot{\zeta} \tag{25}$$

$$\ddot{\xi} = \xi''\dot{\zeta}^2 - \eta\xi'\dot{\zeta}^2 \;* \tag{26}$$

leading to the transformed equation for (8)

$$\xi'' = g_z/\dot{\zeta}^2 = g_z\, v_{xb}^{-2}\, e^{2\eta\zeta} \quad . \tag{27}$$

This equation can be integrated directly to yield

$$\xi' = g_z(e^{2\eta\zeta} - 1)/[2\eta v_{xb}^2] + \xi'(t_b) \tag{28}$$

where

$$\xi'(t_b) = \dot{\xi}/\dot{\zeta}|_{t = t_b} = v_{zb}/v_{xb} \tag{29}$$

and

$$v_{zb} = V_{zb} - w_z \quad . \tag{30}$$

A second integral yields

$$\xi = g_z(e^{2\eta\zeta}-1)/[4\eta^2 v_{xb}^2] - g_z\zeta/[2\eta v_{xb}^2] + v_{zb}\zeta/v_{xb} \quad . \tag{31}$$

*NOTE: Here g_x has been dropped.

This equation is the solution for the normalized drop ξ in terms of normalized range ζ. To change the dependence of ξ from ζ to t one can proceed as follows: Observe that in Eq. (23) the dominant part of the solution can be written

$$t - t_b = (\eta v_{xb})^{-1}(e^{\eta\zeta}-1) \tag{32}$$

since λ is small for launch conditions having small angles with respect to horizontal. This can be viewed as a first order solution to Eq. (23). To incorporate the λ dependence let

$$e^{\eta\zeta} = 1 + \gamma \tag{33}$$

where

$$\gamma = \eta v_{xb}(t-t_b) \quad . \tag{34}$$

Also by squaring and cubing Eq. (33)

$$e^{2\eta\zeta} -1 = 2\gamma + \gamma^2 \tag{35}$$

$$e^{3\eta\zeta} - 1 = 3\gamma + 3\gamma^2 + \gamma^3 \quad . \tag{36}$$

From Eq. (23) one has

$$e^{\eta\zeta}-1 = \gamma + (\lambda/6)(e^{3\eta\zeta}-1) = \gamma + (\lambda/6)(3\gamma + 3\gamma^2 + \gamma^3). \tag{37}$$

By squaring Eq. (37) one obtains

$$e^{2\eta\zeta}-1 = 2\gamma + \gamma^2 + \lambda(\gamma + 2\gamma^2) + \text{term of order } (\lambda\gamma^3) + \text{term of order } (\lambda^2). \tag{38}$$

By use of Eq. (38) (without the higher order terms in Eq. (31) one obtains

$$\xi = g_z(2\gamma+\gamma^2+\lambda\{\gamma+2\gamma^2\})/[4\eta^2 v_{xb}^2]-g_z\zeta/[2\eta v_{xb}^2]+v_{zb}\zeta/v_{xb} \quad . \tag{39}$$

By restoring all variables to their original form and observing that

$$\zeta v_{zb}/v_{xb} = V_{zb}\zeta/v_{xb} - w_z\zeta/v_{xb} \quad , \tag{40}$$

Eq. (39) takes the form

$$z = z_b + w_z\,(t - t_b - \zeta/v_{xb}) + \frac{g_z}{2\eta v_{xb}}\,(t - t_b - \zeta/v_{xb})$$

$$+ \frac{1}{4}\,g_z\,(t - t_b)^2 + \frac{g_z\lambda(t-t_b)}{4\eta v_{xb}} + \frac{g_z\lambda(t-t_b)^2}{4} + V_{zb}\,\zeta/v_{xb} \quad . \tag{41}$$

This equation provides a precise solution form for analyzing the deficiencies, omissions and correctness/or not of the BHT free flight components of the COBRA rocket ballistic algorithm.

III. COMPONENT ANALYSIS OF THE GENERAL SOLUTION

z is the so-called drop of the rocket in a vertical plane normal to the initial rocket line; z_b is the drop which occurs during the rocket boost phase.

The term $w_z\,(t - t_b - \zeta/v_{xb})$ is the effect caused by a wind normal to the rocket line in free flight.

Except for the coefficient of 1/4, the term 1/4 $g_z(t - t_b)^2$ corresponds to the free fall of an object in vacuum. The term $[g_z/2\eta v_{xb}](t - t_b - \zeta/v_{xb})$ represents the change in free fall due to drag. It is important to note that as drag becomes very small

$$t - t_b \rightarrow \zeta/v_{xb} \quad . \tag{42}$$

Also as drag approaches zero the quantity η also approaches zero so that the term as a whole becomes indeterminate. It can be shown that

$$\lim_{\eta\to 0} \frac{g_z}{2\eta v_{xb}}\,[t - t_b - \zeta/v_{xb}] = \frac{g_z}{4}\,(t - t_b)^2 \quad . \tag{43}$$

This result is obtained by expanding the exponential in Eq. (32) in a Taylor series and inserting the result in Eq. (43). Accordingly as η approaches zero the sum of the two terms approach 1/2 $g_z\,(t - t_b)^2$ as expected.

The two terms 1/4 $g_z\lambda(t - t_b)^2$ and $[g_z\lambda(t-t_b)]/4\eta v_{xb}$ are interactions between drop in the z and x directions. For flat trajectories that are nearly horizontal these terms are relatively small. Through regression analysis it has been determined that 1/4 $g_z\lambda(t-t_b)^2$ is the more significant term and the remainder can be dropped for the 2.75 inch rocket at ranges limited to 6 km or less.

The term $V_{zb}\ \zeta/v_{xb}$ is closely related to the burnout angle of the velocity vector. Note that

$$\zeta = x - x_b - w_x\ (t - t_b) \tag{44}$$

and

$$v_{xb} = V_{xb} - w_x\ . \tag{45}$$

For zero wind along the x direction, $w_x = 0$ and

$$V_{zb}\ \zeta/v_{xb} \rightarrow (V_{zb}/V_{xb})(x - x_b) \approx \theta_b\ (x - x_b) \tag{46}$$

where θ_b is the velocity burnout angle. The availability of this component analysis now makes possible a precise critique of the BHT free flight components.

IV. THE BHT FREE FLIGHT COMPONENT

The BHT algorithm is designed to provide the angular setting for engaging a moving target at a range R having a velocity V_{tar_r} along the line of sight and V_{tar_ε} normal to the line of sight. Warhead effectiveness criteria dictate that the rocket warhead functions at an offset point given by the increments ΔZ and ΔR from the target location. The R' range is defined to be the range between the point of firing and the desired point of warhead event. The latter point is affected by target motion and offset. Hence

$$R' = R + V_{tar_r}\ t + \Delta R \tag{47}$$

BHT defines an angle ε as the total angular drop from effective launch line to impact (warhead event). This angle is given by

$$\varepsilon = \sin^{-1} Z_{BHT}/R' \tag{48}$$

Z_{BHT} is approximated by the equation

$$Z_{BHT} = \cos \Theta_e [Z_b + \Delta Z + C\ (t - t_b)^2] + \theta_b (R' - R_b) - V_{tar_\varepsilon} t + V^*_{AZ}\ t \tag{49}$$

where

$$V_{AZ}^{*} = V_{gw} - V_{gu} \tan \theta \tag{50}$$

The components of V_{AZ}^{*} are defined as follows: u, v, w are body axes components for the helicopter. u is forward along the armament datum line (ADL), v is through the right wing and w points down (normal to both u and v). The subscript g refers to velocity with respect to ground, then resolved in this u, v, w system. θ is the angle between the ADL and horizontal. V_{gw} and V_{gu} are helicopter velocity components resolved as described above.

By comparing Eq. (49) with Eq. (41) and in light of the component analysis the following major differences are evident.

A. The BHT equation has no drag term. In an early version of Ref. 1, C was set to g/2. Later it was adjusted to a value of 4.1 (= .84 g/2).

B. There is no compensation for wind normal to the launch line in free flight. (Absence of $w_z\,(t - t_b - \zeta/\nu_{xb})$ or equivalent term).

C. The $V_{AZ}^{*}\,t$ term seems to be "ad hoc" in that it was added without physical justification or fitted correlation. The conjecture here is that BHT reasoned (faultily?) as follows: [The vertical component of helicopter velocity is given approximately by

$$V_{AZ}(\text{vert}) \simeq V_{gw} - V_{gu} \tan \theta \quad . \tag{51}$$

This velocity at launch is effectively imparted to the burnout velocity and the decrement in velocity remains throughout free flight, leading to a total decrement in vertical displacement given by $V_{AZ}^{*}t$.] (End of conjecture.) Validated ballistic theory suggests that the effect of any such velocity component is the so-called "weathervaning" effect. The imparted velocity at burnout is of sign opposite to the effective wind and its magnitude is multiplied several fold. More correctly it should enter through the quantity V_{zb} contained in Eq. (41). That equation shows that the general solution depends on the initial z velocity (the burnout component V_{zb}) only through the term v_{zb}. Hence a z component of velocity such as V_{AZ}^{*} at launch in inertial space leads to a z component V_{Azb}^{**} at burnout which as noted above is quite different from V_{AZ}^{*} due to weathervaning. The free flight effect of V_{AZ}^{*} at launch leads to addition of a term given by

$$V_{Azb}^{**}\,\zeta/v_{xb} = V_{Azb}^{**}\,t\,(\zeta/v_{xb}t) \simeq V_{Azb}^{**}\,t\,(\zeta/\zeta_{vacuum}) \tag{52}$$

which arises directly from Eq. (41). Hence, in addition to the difference between V^{**}_{Azb} and V^{*}_{Az}, the expected effect differs by the above noted range ratio.

In any case the effect of initial z velocity can only be obtained by a self consistent fitting of data which includes weathervaning influence in addition to any translational effects. The ad hoc addition to a formula of such a translational term after fitting can only be done correctly if the data did not contain the translational effect. <u>The term in question is believed to be a significant source of error in the COBRA rocket ballistics.</u>

D. <u>There is no compensation in the time of flight equation for range winds.</u> As a result there is no compensation in the sight settings for the effect of range winds since the free flight influence would correctly enter through the t terms in Eqs. (41) and (49). The audit trail of this historical omission reveals the following. The source of the BHT time of flight equation is the BRL document, Ref. 6. That equation was developed under the assumption of no wind. <u>It did not include a wind correction.</u> That equation <u>does</u> contain a <u>correction for non-zero helicopter velocity</u> V_a. <u>That</u> correction takes the form

$$\Delta t = A_5 \rho^2 (R-R_b)^3 \; V_a/V_b \tag{53}$$

where ρ is air density, V_a is helicopter speed, and V_b is burnout velocity. A_5 is a fitting constant. A telephone conversation with Laird Taylor, formerly of BHT, reveals the following. Taylor indicated that the time of flight equation was fitted to trajectory data which included significant influence of wind. The above term was modified by replacing V_a with V_{au} where

$$V_{au} = V_{gu} - w_u \; . \tag{54}$$

BHT apparently believed that one could make the time of flight equation applicable to wind conditions by merely replacing the ground speed V_a with the air speed V_{au}. For example, BHT in Ref. 2, page 4-3 states that "For rockets, wind is implicitly contained within the BRL equation." This is not correct. The dominant effect of wind is corrected for by the general solution for t through the definition of ζ and v_{xb}. For more details see, Ref. 5. <u>The omission of wind correction for time of flight in free flight is a major source of error in the BHT COBRA rocket ballistic algorithm.</u> (Note for the MK 66 with submunition warheads, warhead event is based on fuze setting. This fuze setting can be in error by about .5 seconds for a 20 knot range wind) .

6. *Letter from BRL to USA MICOM (Mr. Bergman), SUBJECT: "Time of Flight Equations for 2.75 Inch Rocket," 22 Dec 1976.*

E. The BHT equations do not compensate for cross winds in free flight. This observation follows immediately from considerations parallel to the above discussion. The displacement in impact prediction (error) is closely approximated by $w_y(t-t_b-\zeta/v_{xb})$ where w_y is the cross wind.

F. The term $V^*_{Ay}\,t$ is symmetrical in structure to $V^*_{Az}\,t$ and is also a significant source of error.

G. Through conversations with BHT personnel it is apparent that the weathervaning term for the azimuth equation (for the MK 40) was field adjusted to "correct" for wind effects showing a bias. In the absence of a predictor for the free flight wind effect this field adjustment is probably in error. The problem is compounded due to symmetry since the adjustment thereafter (and currently) also is included in the equation for the elevation angle.

V. NUMERICAL ANALYSIS OF THE DEFICIENCIES

The numerical effect of the deficiencies related to drag and wind effects in free flight can be studied by correlation of trajectory data. Table 1 is a listing of trajectory conditions for an extensive file of MK 40 data at BRL. The free flight portion of this trajectory data can be correlated with Eq. (41). This equation can be rewritten in the form

$$f = V_{zb}/V_{xb} - (v_{xb}/V_{xb})\{(z-z_b) - w_z(t-t_b-\zeta/v_{xb})\}/\zeta$$

$$= A_1(v_{xb}/V_{xb})\frac{(g_z/\eta v_{xb})}{\zeta}(t-t_b-\zeta/v_{xb}) + A_2\frac{(v_{xb}/V_{xb})}{\zeta}g_z(t-t_b)^2 \ . \qquad (55)$$

The following should be noted in this equation. The quantity V_{zb}/V_{xb} is essentially the burnout angle of the velocity vector. The quantity v_{xb}/V_{xb} is nearly unity since

$$v_{xb}/V_{xb} = (V_{xb}-w_x)/V_{xb} \quad \text{and} \quad V_{xb} \gg w_x \ . \qquad (56)$$

The quantity $(z-z_b)/\zeta$ is an angle associated with the target height z. The quantity $w_z\ (t-t_b-\zeta/v_{xb})$ is a drift term associated with vertical wind. The equation is dependent on range winds due to t correctly representing the effect on time of flight of a range wind and on the definition of ζ, i.e.,

$$\zeta = x - x_b - w_x\ (t - t_b) \ . \qquad (57)$$

It can be shown that the BHT rocket algorithm is equivalent to the above equation with all wind effects absent and with $A_1 = 0$.

To study the effect of the individual deficiencies the data referred to above was correlated under various assumptions. In Figure 1 the correlation is first performed with all wind terms present but only the t^2 term for the gravity drop. The sliding bias is evident. The single term cannot do the job for all ranges. Residuals are biased positive for short ranges and negative for long ranges. The addition of the second term in Figure 2 removes the bias and greatly tightens the correlation. Figure 3 shows the effect of the single term correlation when the wind components of the model are removed as in the case with the BHT algorithm. The maximum error increases from 16 mils to 38 mils.

Figure 4 shows the effect of utilizing the coefficient of the t^2 term as is currently used in the modernized COBRA for the MK 40. The correlation (without adjustment) is excellent for ranges less than 3 km but thereafter a growing bias is encountered. At 6 km the bias is about 50 mils with a maximum error of 60 mils. Figure 5 indicates that when winds are also included the bias at 6 km is about 60 mils with a maximum error of 80 mils.

The deficiencies cited in the BHT free flight model can be overcome by utilization of the approximation represented by Equation (41) restructured into the form of Equation (55). The correlation obtained with the three dominant terms is shown in Figure 6. The RMS and maximum error are reduced by a factor of 17 from that depicted in Figure 5.

It should be noted that the above analysis is contingent on all boost phase effects being perfectly represented and that no "synergistic" interactions arise during the iterations. As shown herein there are additional problems with the algorithm due to poor boost phase modeling. This of course compounds the problem.

VI. AN ANALYSIS OF PAST ROCKET FIRING TESTS

Conversations with test directors and technical personnel associated with past firings of the 2.75 inch rockets from the COBRA, AH-1S indicate that firings show no obvious bias. This is in contrast to the inferences made herein. Some shots (means of a group) are long and others are short - as to be expected from such an inaccurate weapon system. Despite these past cursory observations the analysis performed herein and the audit trail of modeling decisions made by BHT lend insight which leads to the discovery of a large bias in ballistic performance heretofore undetected. As will be shown below, this bias is as high as 60 mils in elevation at a range of 6 kilometers. This in turn leads to a bias in rocket impact of over 400 meters.

Sections III, IV and V showed that the BHT modeling of gravity drop ignores a very significant effect arising from drag. Additionally, a constant in the weathervaning term was empirically adjusted to match test data. Such an adjustment of one constant could only produce a compromise since three significant defects exist. The third, in addition to the drag and weathervaning, is that related to the initial condition effect for velocity components

normal to the line of sight. Intuition suggests that the empirical adjusting of the weathervaning constant of necessity also adjusted for the mismodeling of the other two effects. This implies that for hover firings, where weathervaning is generally insignificant, test firings would show a bias due primarily to the mismodeling of gravity drop.

Test data for 2.75 inch MK 40 rocket firings employing the BHT algorithm are contained in TECOM reports by Andrese[7] and Sanborn[8]. These reports convert miss distance to an equivalent elevation error. As a result of insight provided by the above discussion the data for hover firings were examined in search of the inferred bias. Test data are superimposed upon the elevation error predictions shown in Figure 5. The test data provide conclusive confirmation of the bias inferred from the theoretical ballistic analysis employed in the critique. The bias at 6 km is approximately 60 mils, but as inferred by the analysis the actual error can be as large as 80 mils due to the combination of effects when wind is also included.

VII. CONCLUSIONS

The accuracy of the Cobra 2.75 inch rocket systems is needlessly reduced by deficiencies in basic equations in the ballistic algorithm. A question concerning the adequacy of changes made by Bell Helicopter Textron to BRL provided ballistic equations is largely answered by theoretical analysis and confirmation utilizing test data. Deficiencies such as the lack of a drag component, lack of wind corrections in free flight, and incorrect compensation for helicopter velocity normal to the line of sight can lead to as much as 80 mils elevation error ($\approx$ 600 meters range error) at maximum range for the currently implemented MK 40. Use of the structure of the existing algorithm for the MK 66 would needlessly reduce the system accuracy for that rocket family. A means for correcting most of the deficiencies is available and currently being implemented through the analysis efforts recently performed at BRL in support of the AAH (Apache) development.

---

[7] J. Andrese, Final Independent Evaluation Report for the Enhanced Cobra Armament Program (ECAP) of the AH-1S Modernized COBRA, US Army Test and Evaluation Command, APG, MD, Sep 1980.

[8] J. Sanborn, Final Report of First Article - Preproduction Test (FA-PPT) of the AH-1S Modernized COBRA, TECOM Project No. 4-AI-100-01S-017, Oct 1980.

TABLE 1. MK 40 (M151) TRAJECTORY DATA FILE

(FILE 2755)

DATA BASE OF SIX DEGREE OF FREEDOM TRAJECTORY CALCULATIONS

(MK 40 2.75 " Trajectories)

VA (Knots)	DOWNWASH VELOCITY (M/S)*	ρ (% of Std)	W_1 (Knots) Range Wind	W_3 (Knots) Cross Wind	DIVE Deg.	ALT of Hel. M	I.C. (Code)
-45	0	+10%	0	10	0	15.2	01
-20	0	Std.	10	-10	-5	500	02
0	13.716	-10%	0	0	-10	1000	03
15	6.858	Std.	0	0	0	2000	04
30	0	+5%	-20	-10	+5	15.2	05
50	0	-5%	20	-20	+10	100	06
100	0	Std.	-10	0	-10	1000	07
150	0	Std.	0	20	+10	15.2	08
200	0	Std.	0	10	0	250	09
8	10.058	+10%	0	-20	10	750	10
23	3.200	-10%	0	-20	-10	1500	11
-30	6.858	Std.	10	0	- 5	1000	12
-15	6.858	+10%	20	0	0	500	13
0	6.858	-10%	-10	0	5	500	14
15	6.858	Std.	-20	0	10	50	15

All QE's print every two seconds plus impact at ground.

*For all cases downwash is directed along the local gravity vector regardless of the aircraft attitude. Downwash is treated as a step function with the listed velocity of downwash as amplitude and acts over a fixed distance of 6.279 meters.

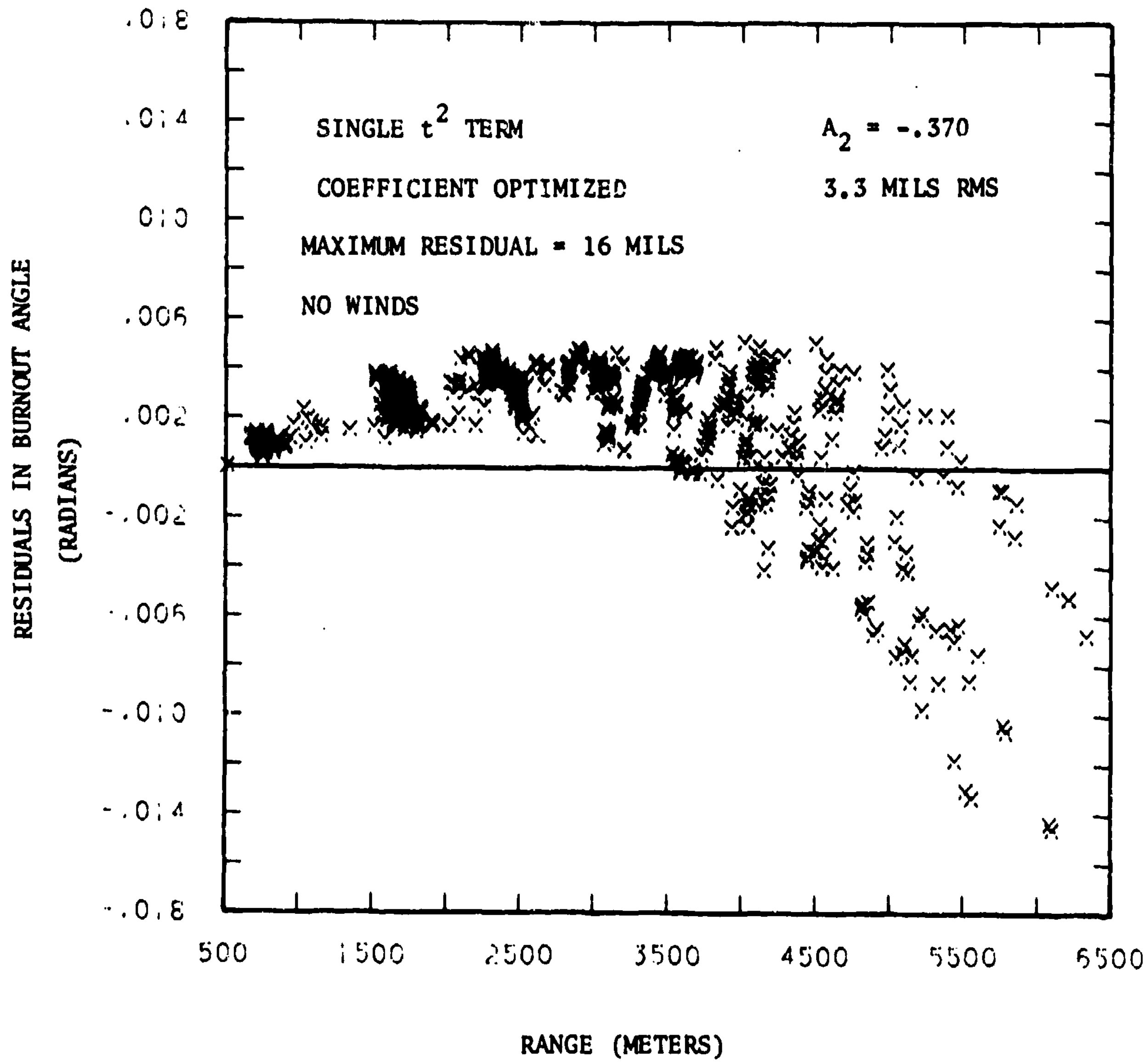

Figure 1. Correlation of MK 40 trajectory data with single t^2 term for gravity drop and coefficient optimized by BRL.

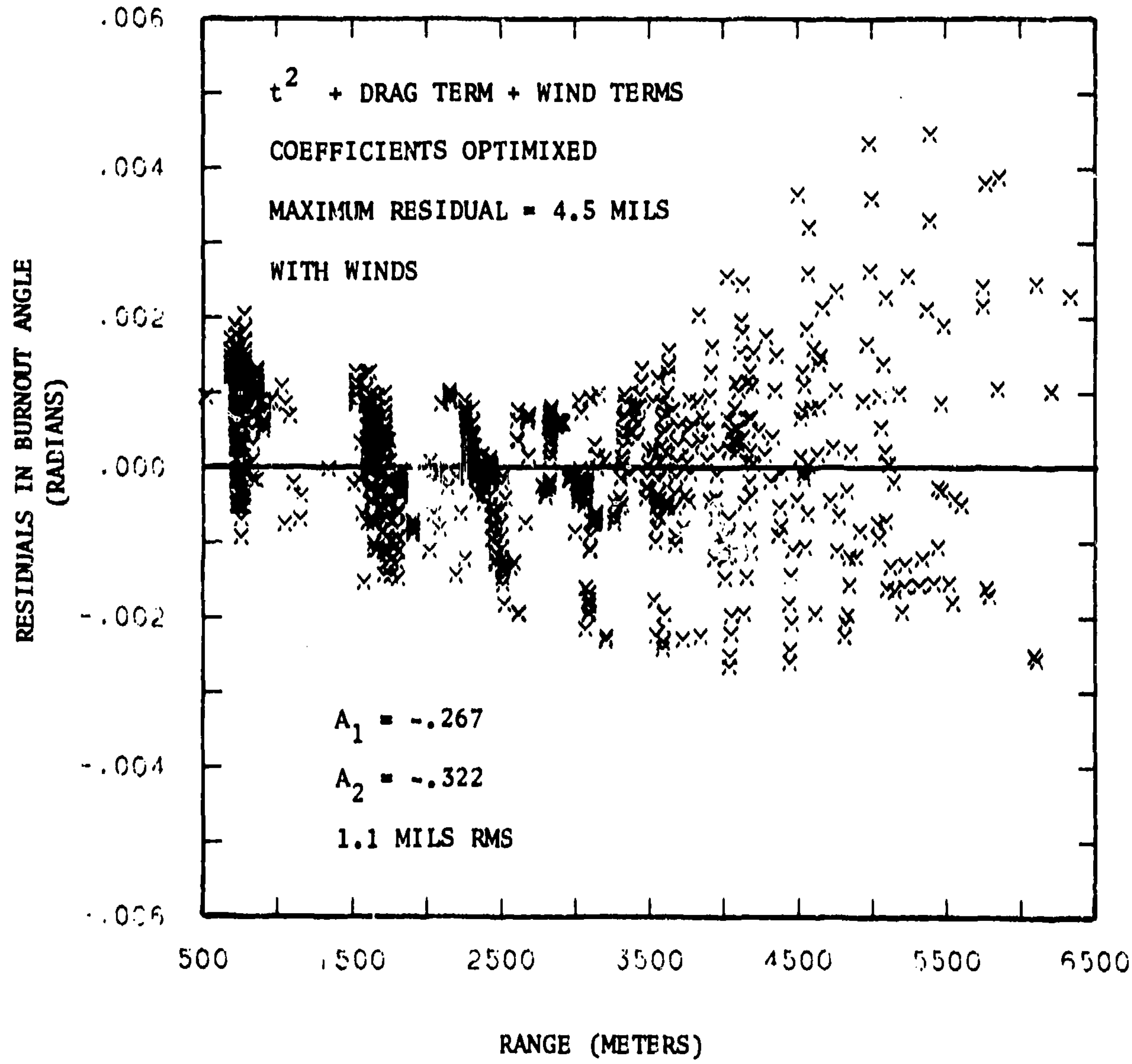

Figure 2. Correlation of MK 40 trajectory data with t^2 term, wind terms and drag term as arising from BRL 80 theory.

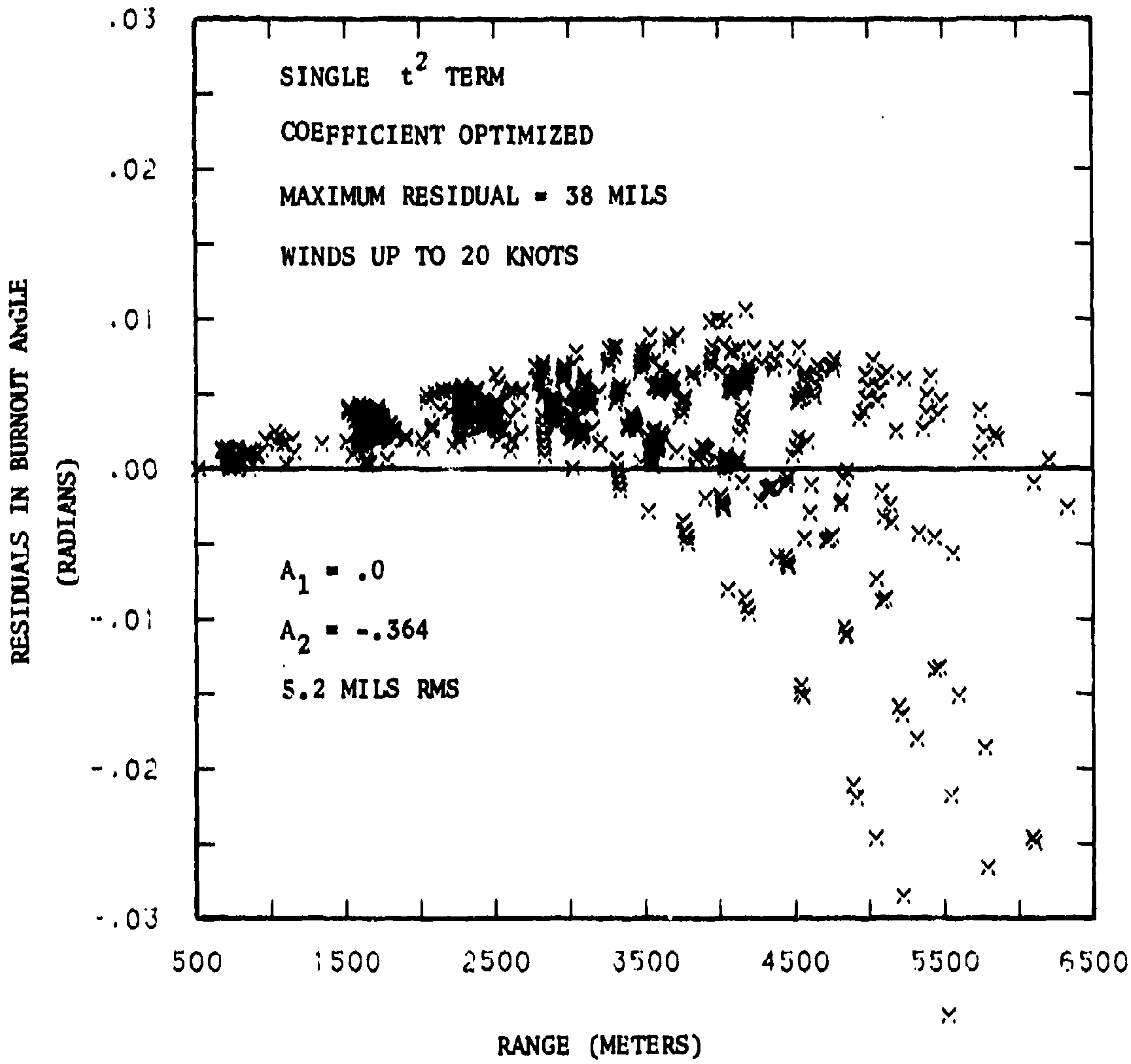

Figure 3. Correlation of MK 40 trajectory data with single t^2 term for gravity drop, wind not compensated for, and coefficient optimized.

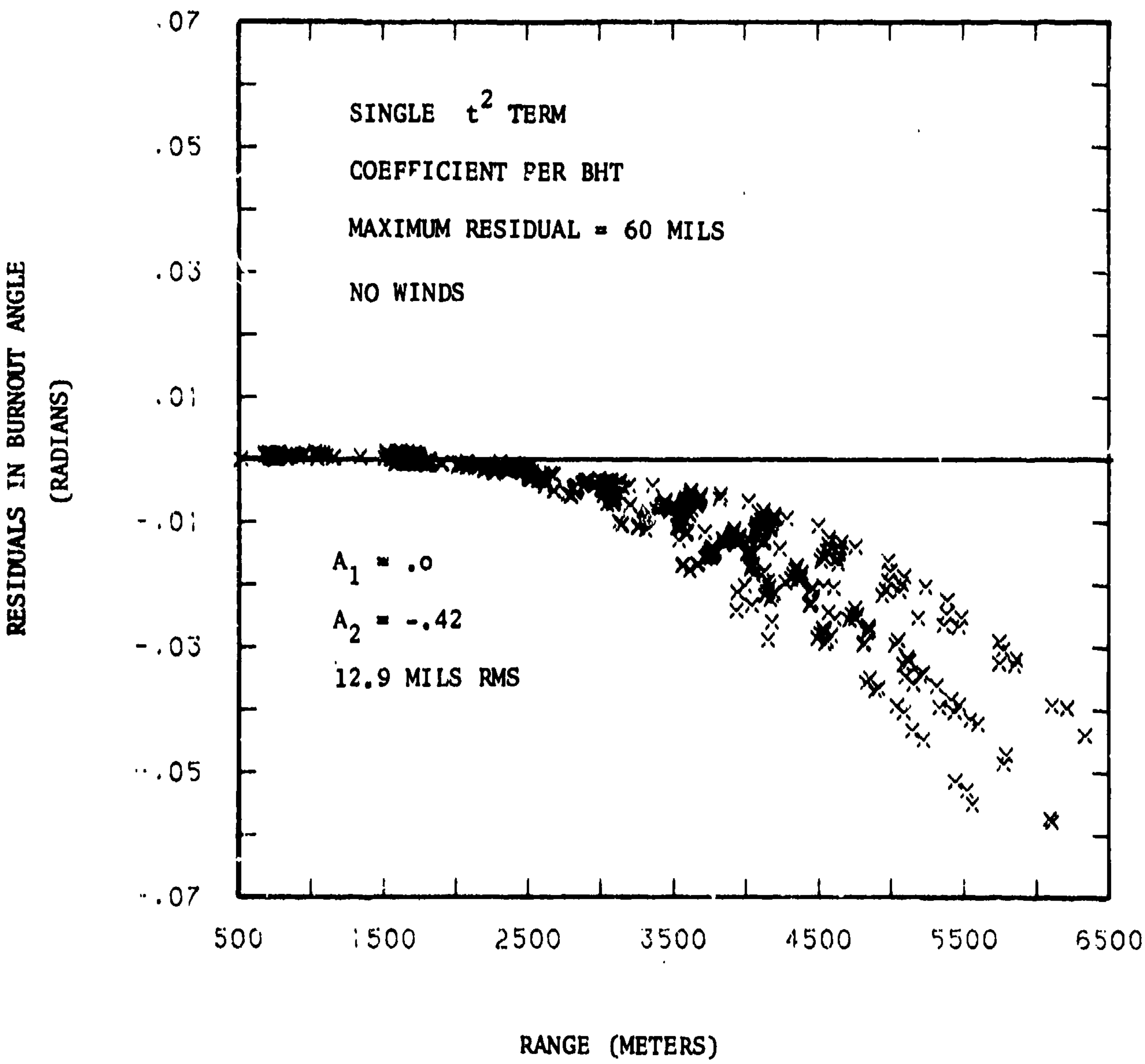

Figure 4. Correlation of MK 40 trajectory data with single t^2 term for gravity drop and coefficient as chosen by BHT.

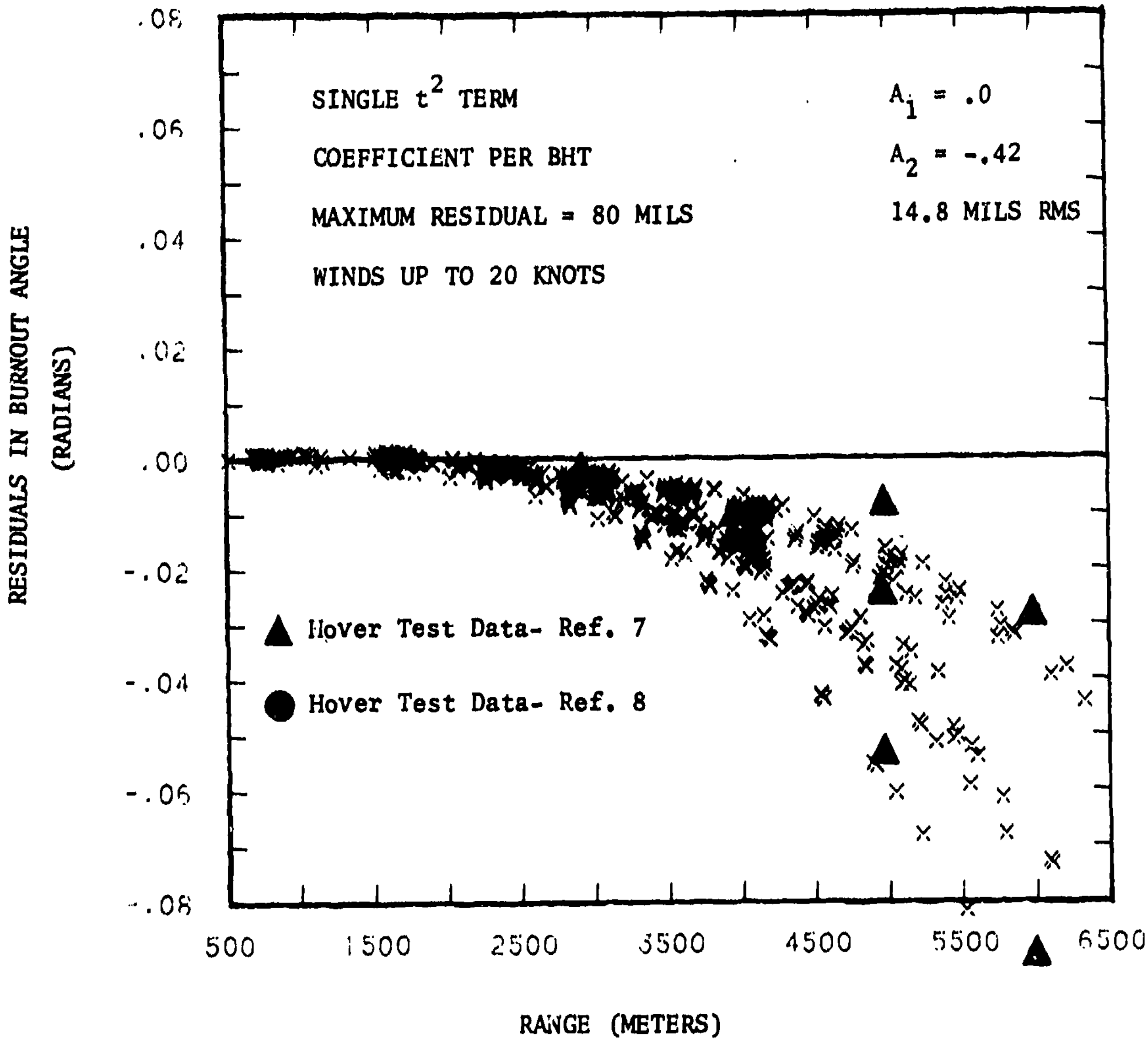

Figure 5. Correlation of MK 40 trajectory data with single t^2 term for gravity drop, wind not compensated for, and coefficient as chosen by BHT. (Test data superimposed.)

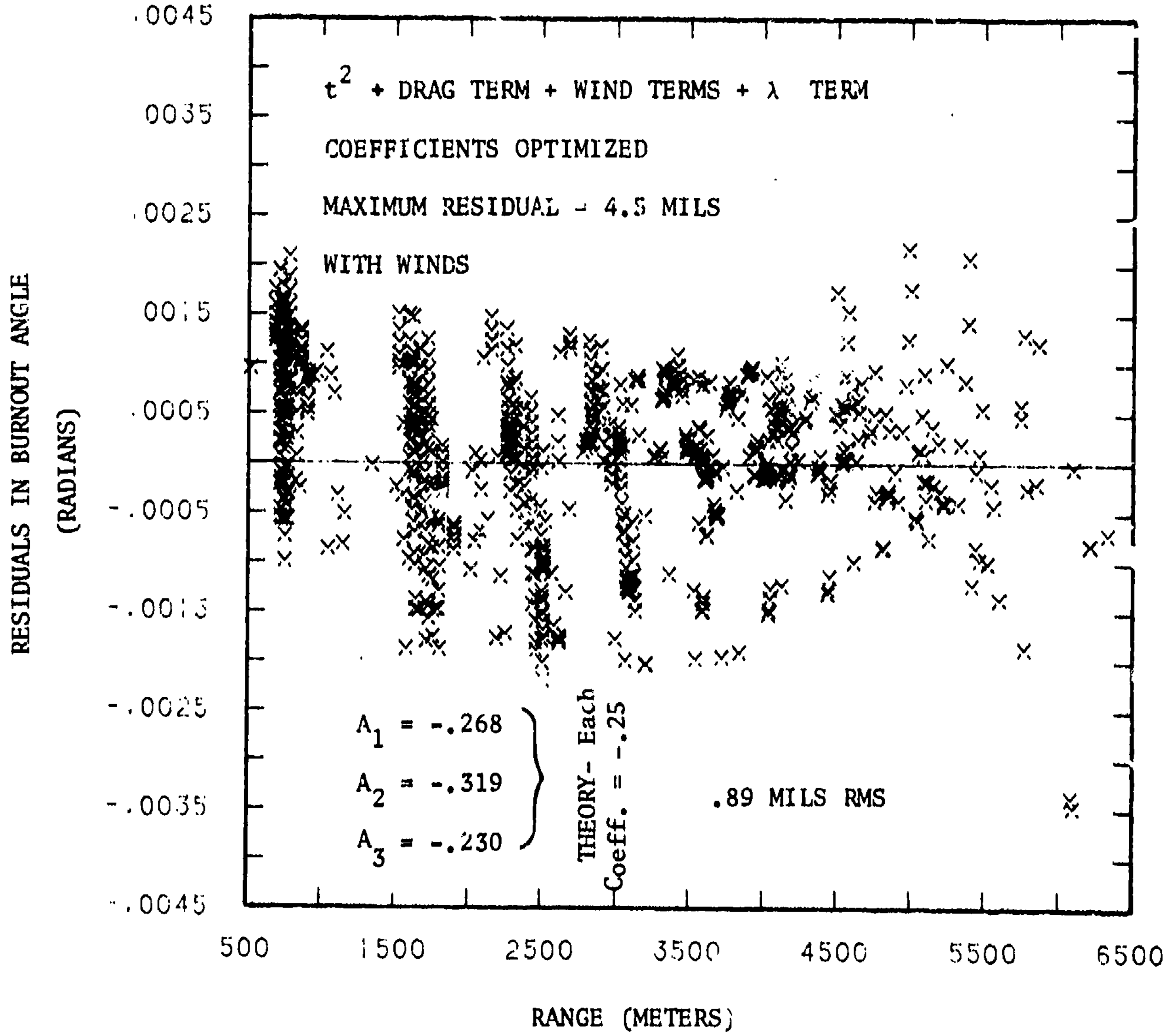

Figure 6. Correlation of MK 40 trajectory data against the complete free flight model arising from BRL 80 theory.

REFERENCES

1. Bell Helicopter Textron Report No. 209-099-520 (07/20/78), "Modernized COBRA Fire Control Computer Mechanization Equations," Rev. B, 15 Aug. 1978.

2. Ibid, Rev E, 22 Aug. 1979.

3. Clark, David L., "A Sight Setting Equation for Air Launched Ballistic Rockets," Unpublished BRL manuscript.

4. O'Bryon, James F., "Numerical Comparison of BHT Equation #209-099-520 With Comparable BRL Equation for the MK 40 2.75 Inch Rocket," contained in a ltr dtd 20 Sep. 1978 to PM COBRA (Mr. Ryan), SUBJECT: Fire Control Equations Data for 2.75 Inch Rockets for AH-1S Modernized COBRA.

5. Breaux, Harold J., "A Methodology for the Development of Fire Control Equations for Guns and Rockets Fired From Aircraft," April 1982, Draft BRL Manuscript.

6. Letter from BRL to USA MICOM (Mr. Bergman, SUBJECT: "Time of Flight Equations for 2.75 Inch Rocket," 22 Dec 1976.

7. J. Andrese, Final Independent Evaluation Report for the Enhanced Cobra Armament Program (ECAP of the AH-1S Modernized COBRA, US Army Test and Evaluation Command, APG, MD, Sep 1980.

8. J. Sanborn, Final Report of First Article - Preproduction Test (FA-PPT) of the AH-1S Modernized COBRA, TECOM Project No. 4-AI-100-01S-017, Oct 1980.

DISTRIBUTION LIST

No. of Copies	Organization
12	**Administrator** Defense Technical Info Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22314
1	HQDA (DAMA-WSA, LTC Csoka) Washington, DC 20310
1	US Army Material Development and Readiness Command ATTN: DRCDMD-ST 5001 Eisenhower Avenue Alexandria, VA 22333
1	Commander US Army Armament Research and Development Command ATTN: DRDAR-TDC, Dr. D. Gyorog Dover, NJ 07801
2	US Army Armament Research and Development Command ATTN: DRDAR-TSS Dover, NJ 07801
3	Commander US Army Armament Research and Development Command ATTN: DRDAR-LCA-FF Mr. Wasserman Mr. O'Keefe DRDAR-RAP, MAJ Connally Dover, NJ 07801
4	Commander US Army Armament Research and Development Command ATTN: DRDAR-SCF-DA, Mr. Raisner Mr. Clineff DRDAR-SCF-DD, Mr. Dziwak DRDAR-SCS-E, Mr. J. Paz Dover, NJ 07801
1	Director US Army Armament Research and Development Command Benet Weapons Laboratory ATTN: DRDAR-LCB-TL Watervliet, NY 12189
1	Commander US Army Armament Material Readiness Command ATTN: DRSAR-LEP-L Rock Island, IL 61299
1	Commander US Army Aviation Research and Development Command ATTN: DRDAV-E 4300 Goodfellow Blvd St. Louis, MO 63120
3	Commander US Army Aviation Research and Development Command ATTN: DRCPM-AAH-TM, Mr. Jack Lease Mr. M. Corrgiat Mr. Julius Romano St. Louis, MO 63120
1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035
2	Commander Applied Technology Laboratory USARTL (AVRADCOM) ATTN: DAVDL-ATL-ASW, Mr. DiMant Mr. N. Kailos Fort Eustis, VA 23604

DISTRIBUTION LIST

No. of Copies	Organization
2	Commander US Army TSARCOM ATTN: DRCPM-CO, Mr. Krill DRCPM-CO-T, Mr. Ryan P.O. Box 209 St. Louis, MO 63166
1	Commander US Army Communications Rsch and Development Command ATTN: DRDCO-PPA-SA Fort Monmouth, NJ 07703
1	Commander US Army Electronics Research and Development Command Technical Support Activity ATTN: DELSD-L Fort Monmouth, NJ 07703
3	Commander US Army Missile Command ATTN: DRSMI-R DRSMI-RC DRSMI-RDF Redstone Arsenal, AL 35898
2	Commander US Army Missile Command ATTN: DRSMI-HR, Mr. Bergman Mr. Brock Redstone Arsenal, AL 35898
1	Commander US Army Missile Command ATTN: DRSMI-YDL Redstone Arsenal, AL 35898
1	Commander US Army Tank Automotive Rsch and Development Command ATTN: DRDTA-UL Warren, MI 48090
3	Commander US Army Yuma Proving Ground ATTN: STEYP-MAA, Mr. D. Mavis Mr. J. Sanborn Mr. J. Vieyra Yuma, AZ 85364
1	Commander US Army Logistics Evaluation Agency ATTN: DALO-LEI, Mr. Merritt New Cumberland, PA 17070
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL, Tech Lib White Sands Missile Range NM 88002
2	Commander US Army Aviation Center and School ATTN: ATZQ-D-MS, Mr. Wagner ATZQ-D-MT, Mr. Garbarini Fort Rucker, AL 36362
1	Commander Naval Ordance Station ATTN: Code 621, Mr. Bergbauer Indian Head, MD 20640
1	AFELM, The Rand Corporation ATTN: Library-D 1700 Main Street Santa Monica, CA 90406
2	Bell Helicopter Textron BHT - Project Engineer AH-1S ATTN: Mr. Milton Lamm Mr. George Cohill P.O. Box 482 Ft. Worth, TX 76101

DISTRIBUTION LIST

No. of Copies	Organization
3	Hughes Helicopters ATTN: Mr. Donald Price Mr. Dean Zes Mr. R. Watson Bldg. 305 M/S-T73A Centinella Teale St. Culver City, CA 90230
2	Teledyne Corp. ATTN: Mr. Roger Gilreath Mr. Tor Kinsinger 19601 Nordhoff St. Northridge, CA 91324

ABERDEEN PROVING GROUND

Dir, USAMSAA
ATTN: DRXSY-D
DRXSY-MP, H. Cohen
DRXSY-AAG, Mr. Lauzau
Mr. Nicholson
DRXSY-J, Mr. O'Bryon
Cdr, USATECOM
ATTN: DRSTE-TO-F
DRSTE-CT-A, MAJ C. Borns
Mr. J. Farrell
Mr. J. Lee
Dir, USACSL, Bldg E3516, EA
ATTN: DRDAR-CLB-PA

USER EVALUATION OF REPORT

Please take a few minutes to answer the questions below; tear out this sheet, fold as indicated, staple or tape closed, and place in the mail. Your comments will provide us with information for improving future reports.

1. BRL Report Number__________________________________

2. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which report will be used.)

____

____

____

3. How, specifically, is the report being used? (Information source, design data or procedure, management procedure, source of ideas, etc.)____

____

____

4. Has the information in this report led to any quantitative savings as far as man-hours/contract dollars saved, operating costs avoided, efficiencies achieved, etc.? If so, please elaborate.

____

____

5. General Comments (Indicate what you think should be changed to make this report and future reports of this type more responsive to your needs, more usable, improve readability, etc.)__________

____

____

____

6. If you would like to be contacted by the personnel who prepared this report to raise specific questions or discuss the topic, please fill in the following information.

Name:____________________________________

Telephone Number:____________________________________

Organization Address:____________________________________

____

____